Pre-AP Geometry 4-8 Study Guide: Introduction to Coordinate Proof (pp 279-284)

Attendance Problems.

1. Find the midpoint between $(0, 2x)$ and $(2y, 2z)$.

2. One leg of a right triangle has length 12, and the hypotenuse has length 13. What is the length of the other leg?

3. Find the distance between $(0, a)$ and $(0, b)$, where $b > a$.

- I can position figures in the coordinate plane for use in coordinate proofs.
- I can prove geometric concepts by using coordinate proof.

Vocabulary: coordinate proof

Common Core

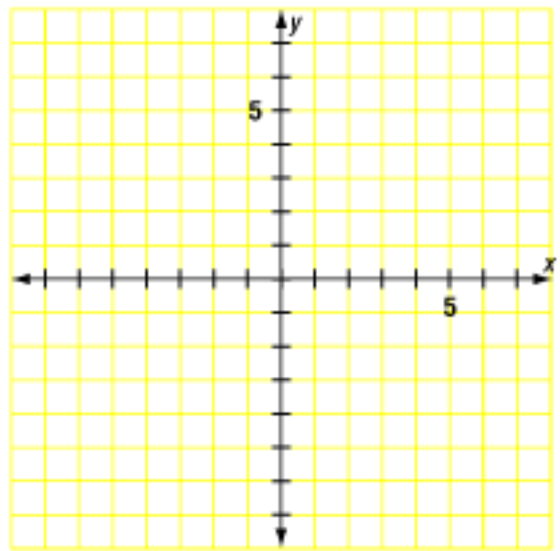
CC.9-12.G.GPE.4 Use coordinates to prove simple geometric theorems algebraically.
CC.9-12.G.GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.*
CC.9-12.G.MG.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*

You have used coordinate geometry to find the midpoint of a line segment and to find the distance between two points. Coordinate geometry can also be used to prove conjectures.
A **coordinate proof** is a style of proof that uses coordinate geometry and algebra. The first step of a coordinate proof is to position the given figure in the plane. You can use any position, but some strategies can make the steps of the proof simpler.

Strategies for Positioning Figures in the Coordinate Plane

- Use the origin as a vertex, keeping the figure in Quadrant I.
- Center the figure at the origin.
- Center a side of the figure at the origin.
- Use one or both axes as sides of the figure.

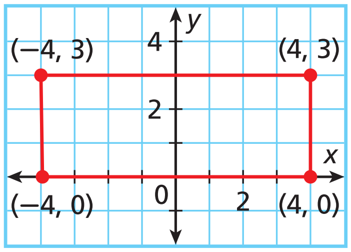
<u>Video Example 1</u>. Position a rectangle with length 4 units and width 6 units in the coordinate plane.



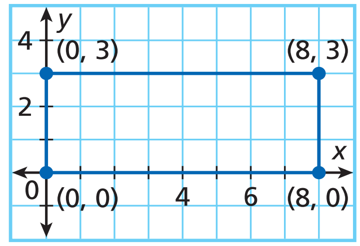
1 Positioning a Figure in the Coordinate Plane

Position a rectangle with a length of 8 units and a width of 3 units in the coordinate plane.

Method 1 You can center the longer side of the rectangle at the origin.

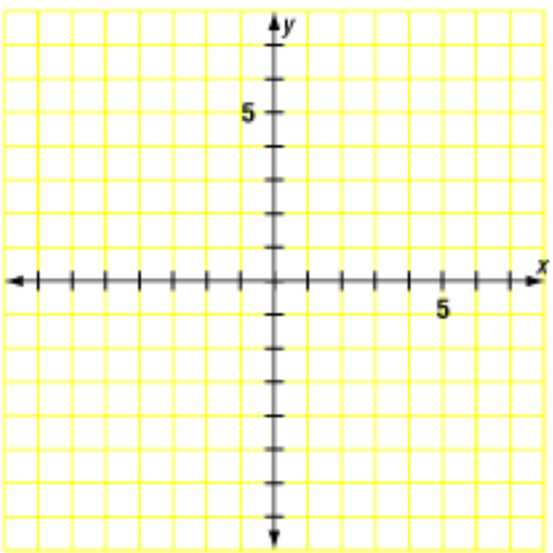


Method 2 You can use the origin as a vertex of the rectangle.

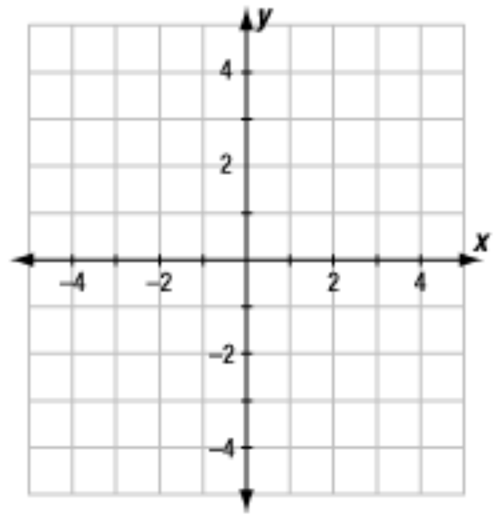


Depending on what you are using the figure to prove, one solution may be better than the other. For example, if you need to find the midpoint of the longer side, use the first solution.

<u>Example 1</u>. Position a square with a side length of 6 units in the coordinate plane.



4. <u>Guided Practice</u>. Position a right triangle with leg lengths of 2 and 4 units in the coordinate plane. (*Hint*: Use the origin as the vertex of the right angle.)

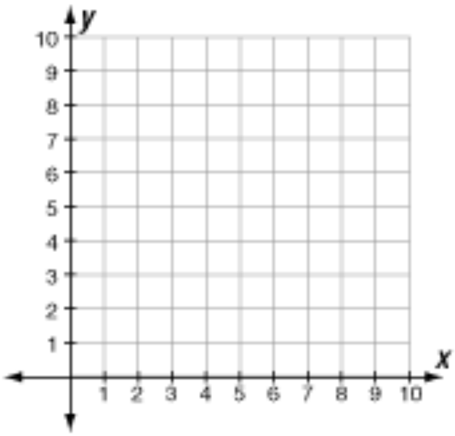


Once the figure is placed in the coordinate plane, you can use slope, the coordinates of the vertices, the Distance Formula, or the Midpoint Formula to prove statements about the figure.

Video Example 2.

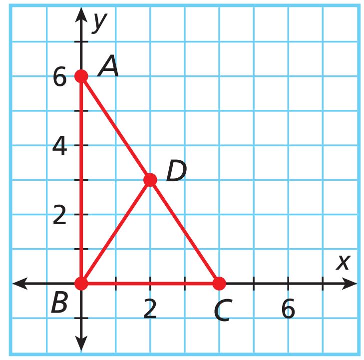
Given: Right $\triangle ABC$ has vertices $A(0,8), B(0,0), \& \; C(5,0)$.
D is the midpoint of $\overline{AC}$.

Prove: The area of $\triangle ABC$ is twice the area of $\triangle DBC$.



2 Writing a Proof Using Coordinate Geometry

Write a coordinate proof.



Given: Right $\triangle ABC$ has vertices $A(0, 6)$, $B(0, 0)$, and $C(4, 0)$. D is the midpoint of $\overline{AC}$.

Prove: The area of $\triangle DBC$ is one half the area of $\triangle ABC$.

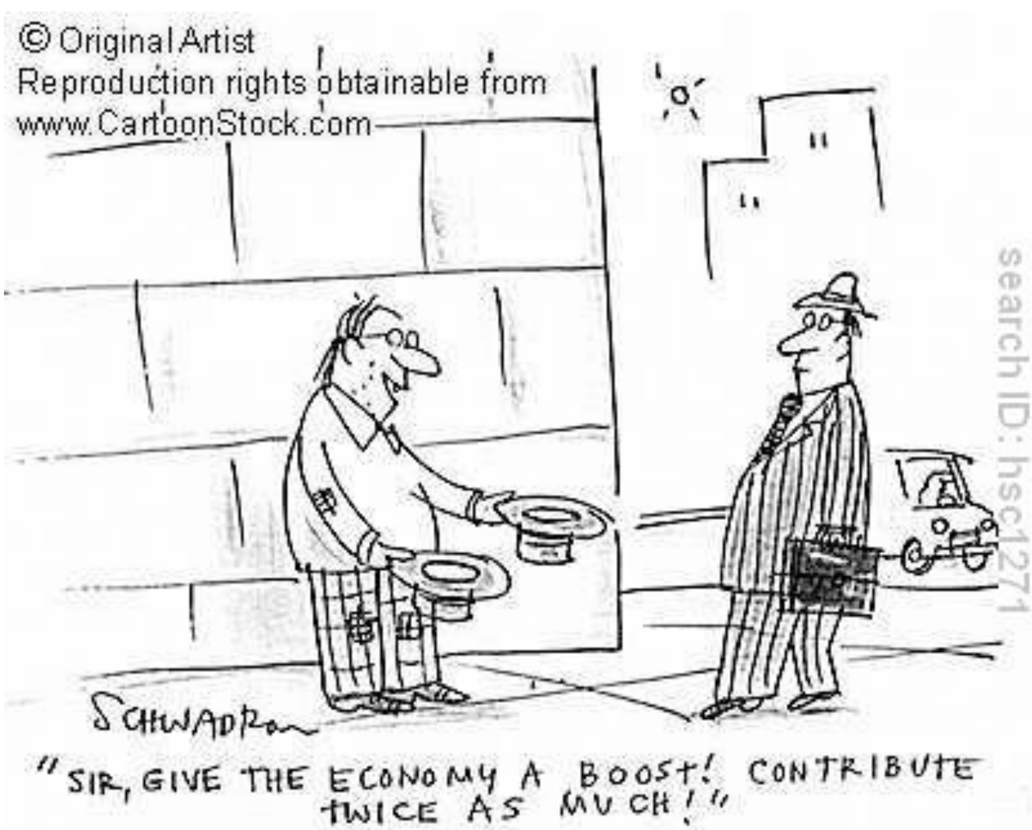
Proof: $\triangle ABC$ is a right triangle with height AB and base BC.

$$\text{area of } \triangle ABC = \frac{1}{2}bh$$
$$= \frac{1}{2}(4)(6) = 12 \text{ square units}$$

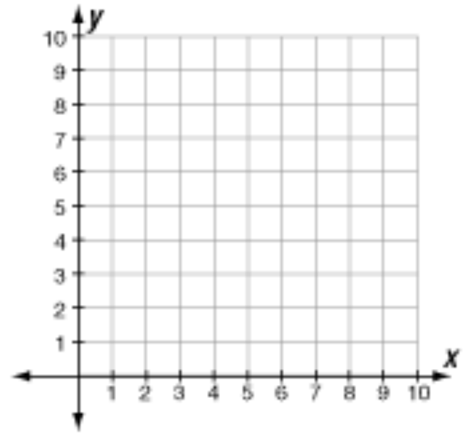
By the Midpoint Formula, the coordinates of $D = \left(\frac{0+4}{2}, \frac{6+0}{2}\right) = (2, 3)$. The y-coordinate of D is the height of $\triangle DBC$, and the base is 4 units.

$$\text{area of } \triangle DBC = \frac{1}{2}bh$$
$$= \frac{1}{2}(4)(3) = 6 \text{ square units}$$

Since $6 = \frac{1}{2}(12)$, the area of $\triangle DBC$ is one half the area of $\triangle ABC$.



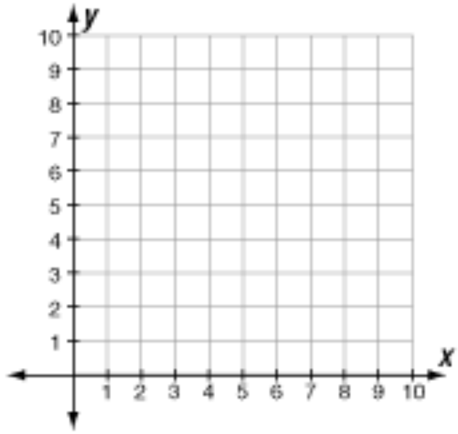
<u>Example 2</u>.
Given: Given: Rectangle *ABCD* with *A*(0, 0), *B*(4, 0), *C*(4, 10), and *D*(0, 10).
Prove: The diagonals bisect each other.



5. <u>Guided Practice</u>.

Given: Right $\triangle ABC$ has vertices $A(0,6), B(0,0), \& \ C(4,0)$.
D is the midpoint of $\overline{AC}$.

Prove: The are of $\triangle ADB$ is half the area of $\triangle ABC$.



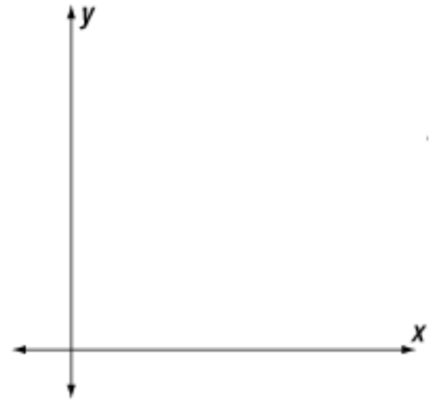
A coordinate proof can also be used to prove that a certain relationship is always true.

You can prove that a statement is true for all right triangles without knowing the side lengths.

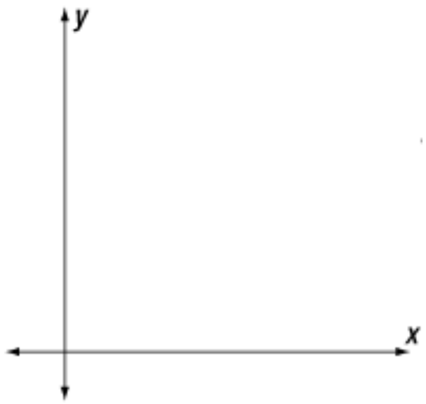
To do this, assign variables as the coordinates of the vertices.

<u>Video Example 3</u>. Position each figure in the coordinate plane and give the coordinates of each vertex.

A. A right triangle with leg lengths m and and n.



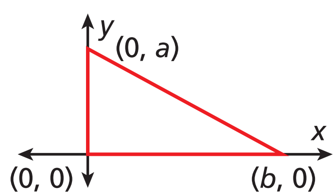
B. A rectangle with length q and width r.



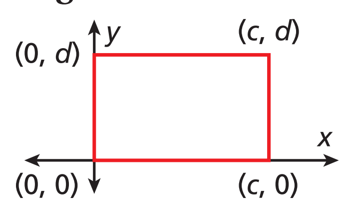
3 Assigning Coordinates to Vertices

Position each figure in the coordinate plane and give the coordinates of each vertex.

A **a right triangle with leg lengths *a* and *b***

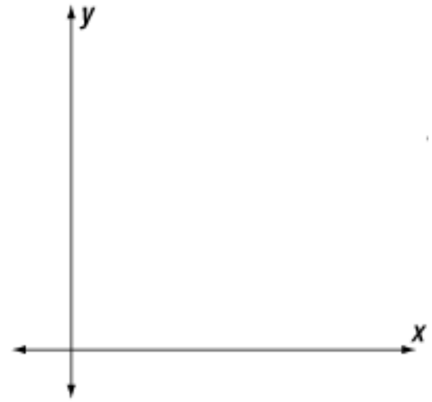


B **a rectangle with length *c* and width *d***

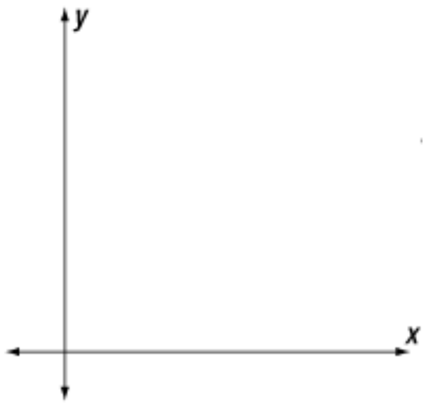


<u>Example 3</u>. Position each figure in the coordinate plane and give the coordinates of each vertex.

A. Rectangle with width *m* and length twice the width.



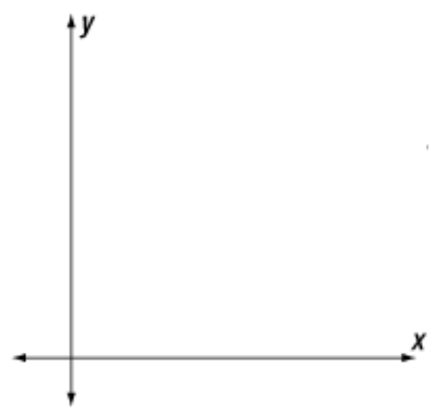
B. Right triangle with legs of lengths *s* and *t.*



Caution!

Do not use both axes when positioning a figure unless you know the figure has a right angle.

6. <u>Guided Practice</u>. Position a square with side length 4*p* in the coordinate plane and give the coordinates of each vertex.



4-8 Introduction to coordinate proof *(pp 282)* 8-12.

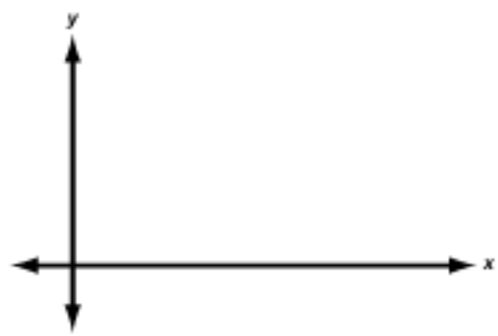
If a coordinate proof requires calculations with fractions, choose coordinates that make the calculations simpler.
For example, use multiples of 2 when you are to find coordinates of a midpoint. Once you have assigned the coordinates of the vertices, the procedure for the proof is the same, except that your calculations will involve variables.

Remember!

Because the *x*- and *y*-axes intersect at right angles, they can be used to form the sides of a right triangle.

Video Example 4.

$\measuredangle B$ is a right angle in $\triangle ABC$.
Given: D is the midpoint of $\overline{AB}$.
E is the midpoint of $\overline{BC}$.
Prove: The area of $\triangle DBE$ is one fourth the area of $\triangle ABC$.



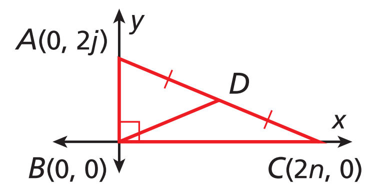
4 Writing a Coordinate Proof

Given: $\angle B$ is a right angle in $\triangle ABC$. D is the midpoint of $\overline{AC}$.
Prove: The area of $\triangle DBC$ is one half the area of $\triangle ABC$.

Step 1 Assign coordinates to each vertex.
The coordinates of A are $(0, 2j)$,
the coordinates of B are $(0, 0)$,
and the coordinates of C are $(2n, 0)$.

Since you will use the Midpoint Formula to find the coordinates of D, use multiples of 2 for the leg lengths.

Step 2 Position the figure in the coordinate plane.



Step 3 Write a coordinate proof.

Proof: $\triangle ABC$ is a right triangle with height $2j$ and base $2n$.

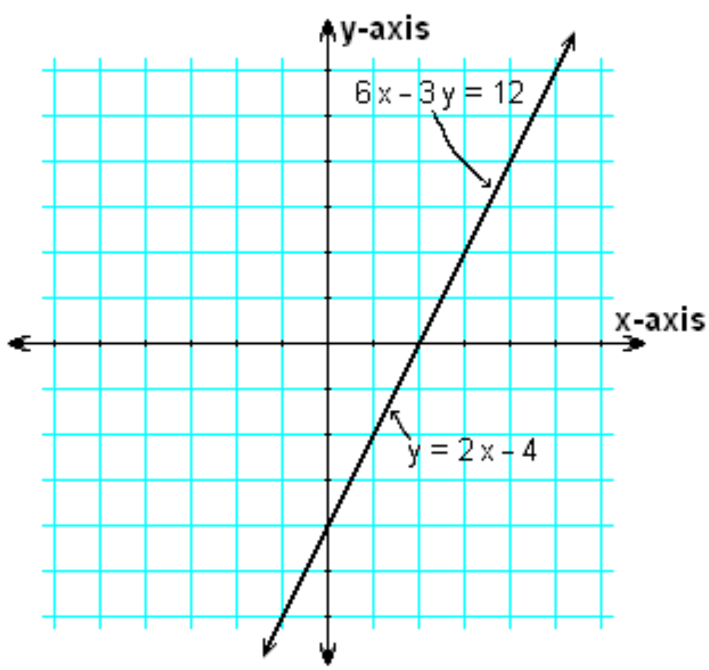
$$\begin{aligned}\text{area of } \triangle ABC &= \tfrac{1}{2}bh \\ &= \tfrac{1}{2}(2n)(2j) \\ &= 2nj \text{ square units}\end{aligned}$$

By the Midpoint Formula, the coordinates of $D = \left(\frac{0 + 2n}{2}, \frac{2j + 0}{2}\right) = (n, j)$.

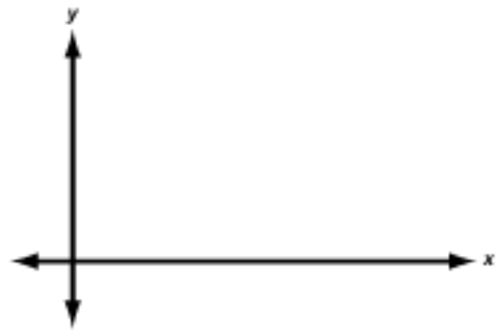
The height of $\triangle DBC$ is j units, and the base is $2n$ units.

$$\begin{aligned}\text{area of } \triangle DBC &= \tfrac{1}{2}bh \\ &= \tfrac{1}{2}(2n)(j) \\ &= nj \text{ square units}\end{aligned}$$

Since $nj = \frac{1}{2}(2nj)$, the area of $\triangle DBC$ is one half the area of $\triangle ABC$.



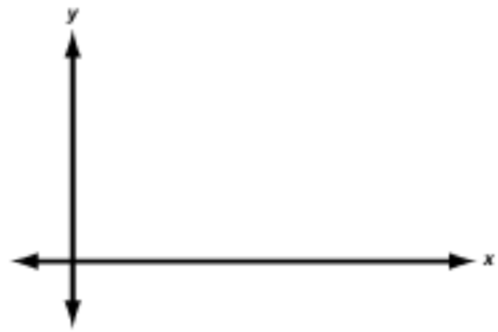
Example 4.
Given: Rectangle *PQRS*
Prove: The diagonals are congruent.



7. <u>Guided Practice</u>.

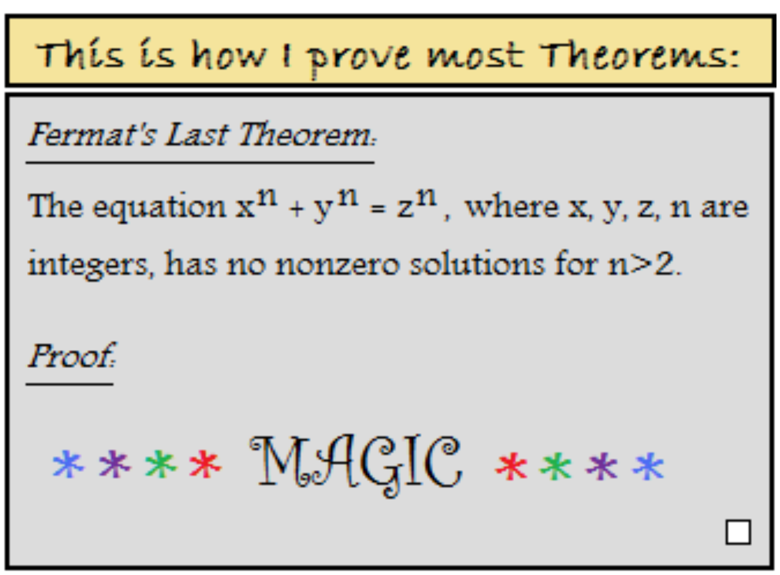
Given: $\measuredangle B$ is a right angle in $\triangle ABC$.
D is the midpoint of $\overline{AC}$.

Prove: The area of $\triangle ADB$ is one half the area of $\triangle ABC$.



4-8 Introduction to coordinate proof *(pp 282)* 8-13, 15.

Q: What do you call a broken angle?
A: A rectangle!



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